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ABSTRACT

The purpose of this exploratory study was to develop a model for evaluating teachers' instructional practices in mathematics and the cognitions associated with these practices. The sample consisted of seven beginning and seven experienced teachers of secondary school mathematics, who each taught one lesson of his or her own design. To evaluate instructional practice, a Phase-Dimension Framework for Assessing Mathematics Teaching was developed. It consisted of three dimensions (tasks, learning environment, discourse) that were adopted from the "Professional Standards for Teachers of Mathematics" of the National Council of Teachers of Mathematics (1991). To evaluate teacher thoughts, a Teacher Cognitions Framework was developed. It also considered teachers' overarching cognitions (goals, knowledge, beliefs) and their cognitions before (planning), during (monitoring and regulating), and after (evaluating and suggesting) their lesson enactments. Data were obtained through observations, lesson plans, videotapes, and audiotapes of structured interviews during the course of one semester. Data analysis suggests that teacher cognitions play a well-defined role in classroom practice. The findings provide useful insights for researchers, supervisors, and teacher educators interested in assessment techniques reflecting recommendations from current reform movements. Three appendixes contain practice ratings of highlighted lessons, a summary chart of lessons dimensions, and a summary of patterns of cognitions. (Contains 1 figure 2 tables, and 74 references.) (Author/SLD)

Evaluation of Instructional Practice in the Secondary School Mathematics Classroom : A Cognitive Perspective

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Abstract

The purpose of this exploratory study was to develop a model for evaluating teachers' instructional practices in mathematics and the cognitions associated with these practices. The sample consisted of seven beginning and seven experienced teachers of secondary school mathematics who each taught one lesson of their own design. To evaluate instructional practice, a Phase-Dimension Framework for Assessing Mathematics Teaching was developed. It consisted of three dimensions (tasks, learning environment, discourse) that were adopted from the *Professional Standards for Teachers of Mathematics* (NCTM, 1991). To evaluate teacher thoughts, a Teacher Cognitions Framework was developed. It consisted of teachers' overarching cognitions (goals, knowledge, beliefs) and their cognitions before (planning), during (monitoring and regulating) and after (evaluating and suggesting) their lesson enactments. Data were obtained through observations, lesson plans, videotapes, and audiotapes of structured interviews during the course of one semester. Data analysis suggests that teacher cognitions play a well-defined role in classroom practice. The findings provide useful insights for researchers, supervisors, and teacher educators interested in assessment techniques reflecting recommendations from current reform movements.

Within the last two decades, the perspective on teaching and learning has shifted from one grounded in behavioral psychology to one grounded in cognitive psychology. Researchers have now broadened their lens of inquiry by moving beyond the mere examination of teacher behaviors to studying teacher cognitions (Shulman, 1986b; Ernest, 1988; Shavelson, 1986). Our purpose in this exploratory study is to examine the relationship between teachers' cognitions and their instructional practice in mathematics. To this end we developed a model that comprises two frameworks. One framework allows for the systematic examination of instructional practice in mathematics using dimensions of lessons (tasks, learning environment, discourse) as articulated in the Professional Standards for Teaching Mathematics (NCTM, 1991). The other framework allows for the study of the full range of teacher cognitions including teacher knowledge, beliefs, and goals across three stages of teaching: preactive (planning), interactive (monitoring and regulating), and postactive (evaluating and suggesting).

In the first section of this paper, we provide a rationale for the development of the model and a description of the two frameworks. This is followed by a description of the methodology used to differentiate the quality of fourteen mathematics lessons and to study the teachers' cognitions associated with these lessons. Next, the data from the two frameworks are integrated from which patterns of instructional practice and cognitions are analyzed. This is followed by a discussion of findings that have implications for researchers and teacher educators for conceiving of mathematics teaching as an integrated whole where cognitions play a well-defined role.

THEORETICAL BACKGROUND FOR DEVELOPMENT OF THE MODEL

Numerous studies conducted within the expert-novice research tradition have yielded consistent findings on the differences in the cognitions and instructional practices of expert and novice teachers (Borko & Livingston, 1989; Leinhardt, 1989; Livingston & Borko, 1990). Recent research using a conception of teaching as problem solving has also shed light on the relationships between cognitions and instructional practice in mathematics (Artzt & Armour-Thomas, 1993; Carpenter, 1989; Fennema, Carpenter & Peterson, 1989).

Some of the components of teacher cognition as it relates to instructional practice include teacher knowledge (Ball, 1991; Peterson, 1988), beliefs (Dougherty, 1990; Peterson, Fennema, Carpenter & Loef, 1989; Richardson, Anders, Tidwell & Lloyd, 1991), goals (Cobb, Yackel & Wood, 1991; NCTM, 1989), and thought processes (Clark & Peterson, 1986; Fogarty, Wang & Creek, 1983; Wilson, Shulman & Richert, 1987). Although such investigations have called attention to the importance of cognitions and behavior in the study of teaching, some conceptual issues have yet to be addressed.

The first problem is the study of components of teacher cognition in isolation of each other. Knowledge, beliefs, goals and thinking processes are presumed to be conceptually intertwined. Thus, studying them in isolation of each other provides an incomplete understanding of the mental life of teachers as it relates to their instructional practice. Some researchers have begun to create frameworks to examine the nature and quality of the interrelationships of these components of teacher cognitions (Fennema et al., 1989; Fennema & Franke, 1992; Shavelson, 1986).

The second problem concerns the absence of a priori criteria against which to judge the quality of instructional practice. Some researchers (e.g., Kagan, 1990; Leinhardt, 1990) have identified this issue as the "ecological validity" or "performance verification" problem in research on teaching from a cognitive perspective. In a recent review of the literature on mathematics teaching practices and their effects, Koehler and Grouws (1992) observed that instructional quality was a topic that researchers have avoided, and recommended that it should be more adequately addressed in research on mathematics teaching.

A third related problem has to do with the narrowness of the range of teaching activities in the classroom about which teacher cognitions have been investigated. Shulman (1986a), Clark & Peterson (1986), and more recently, Brown & Baird (1993) raised these as concerns and recommended a more comprehensive study of the wide variety of teacher cognitions and their relationship to a broader repertoire of teachers' actions in the classroom.

In this study, we have tried to be responsive to these previously unaddressed issues by investigating teacher cognitions and their instructional practice in mathematics. We contend that knowledge, beliefs, and goals directly influence thinking across three stages of teaching: preactive (planning), interactive (monitoring and regulating), and postactive (evaluating and suggesting). These components form a network of overarching cognitions that direct and control the instructional behaviors of teachers in the classroom. From this perspective, we assume that differential instructional practice is conceptually linked to differences in teacher cognitions. (See Figure 1.)

Insert Figure 1 about here

The purpose of this exploratory study is to understand the cognitions of teachers underlying mathematics lessons of differential quality. In order to differentiate the quality of lessons and study teacher cognitions we developed two frameworks. The first one is the Phase-Dimension Framework for the Assessment of Mathematics Instruction (PDF). The second one is the Teacher Cognitions Framework (TCF). A detailed description of the development of each of these frameworks follows.

The Phase-Dimension Framework for the Assessment of Mathematics Instruction

The Phase-Dimension Framework for the Assessment of Mathematics Instruction (PDF) was developed as an instrument for assessing the quality of mathematics lessons through observations. We made an a priori decision to define quality practice as instruction that provides opportunities for students to learn mathematics with understanding. Two issues guided this decision. First, there is a widely shared view among researchers and teachers that the goal of instruction is to promote student learning with understanding (Hiebert & Carpenter, 1992). Secondly, theoretical and empirical research on learning from psychology, mathematics education, and cognitive science suggests positive consequences for students who learn with understanding. For example, some researchers claim that initial understanding enables children to construct relationships and create productive inventions (Hiebert & Carpenter, 1992, Siegler &

Jenkins, 1989; Steffe & Cobb 1988). Others have shown that learning with understanding promotes remembering (Bartlett, 1932, Rumelhart, 1975), and it enhances transfer (Brown, Collins, & Duguid, 1989; Lave, 1988; Perkins & Salomon, 1989).

Within recent years, research from cognitive psychology and mathematics education have provided consistent findings about how students learn with understanding. We believe that these findings have implications for instructional practice. These findings indicate that (a) learning occurs in phases (Jones, Palincsar, Ogle, & Carr, 1987; Lindquist, 1987), (b) it is facilitated when learners link new information to prior knowledge (Fennema et al., 1989; Lampert, 1986), (c) learning is an active problem-solving process (Cobb, 1986; Thompson, 1985), (d) learners transform external information into internal representations (Greeno, 1989; Newell, 1980; Pylyshyn, 1980), (e) learners make connections between and among disparate pieces of information (Chi, 1978; Leinhardt & Smith, 1985), and (f) learners use different cognitive and metacognitive processes in solving complex tasks (Artzt & Armour-Thomas, 1992; Schoenfeld, 1985; Silver, 1985). In drawing implications for classroom practice from the research literature we created two classifications: *Lesson Dimensions* and *Lesson Phases.*

Lesson Dimensions

Lesson dimensions refer to those broad aspects of instructional practice that define critical areas of teachers' work during the enactment of a lesson. We looked for those dimensions of a lesson that suggested areas of instructional practice that might foster student learning with understanding. Toward this end, we selected three dimensions of a lesson: tasks, verbal interactions, and classroom climate. Tasks can provide opportunities for learners to connect their knowledge to new information and to build on their knowledge and interest through active engagement in meaningful problem solving. Verbal interactions can provide opportunities for learners to share experiences that enable them to notice relationships of interest, to justify the relationships they observe and to allow them to assume the responsibility for problem solving. Researchers in teaching and learning mathematics have called attention to the important attributes of tasks and verbal interactions in the classroom that are likely to influence the representations that learners form and the connections they make (Behr, Harel, Post, & Lesh, 1992; Hiebert & Carpenter, 1992; Lampert, 1989; Mack, 1990).

Classroom climate provides a context or ambiance in which students can explore and exchange ideas, work diligently and efficiently on assignments, unfettered by interruptions or distractions. Results from process-product research suggest that a teacher's skill in allocating time for instruction, applying rules for student deportment, pacing, managing routines and transitions have implications for how order is maintained and how the use of instructional time is enhanced (e.g. Armour-Thomas & Szczesiul, 1989, Brophy & Good, 1986). Jones et al. (1987), contend that these management activities provide an advantageous position for strategic teaching with a cognitive focus.

For these dimensions of a lesson we adopted the descriptors set forth by the National Council of Teachers of Mathematics (NCTM, 1991): *Tasks, Learning Environment*, and *Discourse*. Although we are aware that within the classroom setting all of these dimensions are interrelated, we made our best effort to distinguish them through the development of indicators through which they could be evaluated. Each dimension consists of five indicators which, as a cluster, can provide evidence of teaching for the promotion of student learning mathematics with understanding. Each indicator is described by one or more defining attributes that are observable in the classroom. For the dimension *tasks* the indicators are: tools/materials, motivational strategies, content, difficulty level, sequencing. For the dimension *learning environment* the indicators are: social/intellectual climate, administrative routines, instructional routines, pacing, student deportment. For the dimension *discourse* the indicators are: questioning, student responses, teacher-student interaction, teacher responses, student-student interactions. See Table 1 for a list of the indicators and their defining attributes.

Insert Table 1 about here

Lesson Phases

To account for the fact that the quality of instructional practice may, and often does, vary over the course of a lesson, it was necessary to partition the lessons into temporal phases for a more accurate assessment. Lesson phases describe a sequence of the kinds of teaching-learning episodes that unfold over the duration of the lesson. The concept of lesson phases is derived from the cognitive instruction literature that suggests that the way teachers initiate, develop, and close instructional episodes have important implications for student learning (e.g. Costa, 1985; Jones et al., 1987). It appears that phases can establish readiness for learning, enable learners to recognize relationships and construct new meanings and, finally, enable them to integrate and extend their learning to new contexts.

In this study we adopted the descriptors for the temporal phases of a lesson from the Connecticut Competency Instrument (Beginning Educator Support & Training Program, 1989): *Initiation, Development* and *Closure.* We examined the instructional dimensions within each phase of the lesson.

The Teacher Cognitions Framework

The Teacher Cognitions Framework (TCF) was developed to examine the mental activities of teachers associated with instructional practice. We used Jackson's (1968) conceptual distinctions of preactive, interactive and postactive stages of teaching to examine teacher cognitions before, during and after teaching a lesson and have selected eight components of cognition for study: *knowledge, beliefs, goals, planning, monitoring, regulating, evaluating* and *suggesting.*

We define teacher knowledge as an integrated system of internalized information acquired about pupils, content and pedagogy. This definition is based on Shulman's (1986b) conception of teacher knowledge as a multidimensional and interrelated construct that include subject matter knowledge, pedagogical knowledge, and knowledge of students. We concur with the views of other researchers (Fennema & Franke, 1992; Leinhardt, Putnam, Stein, & Baxter, 1991; Peterson, 1988) that these components of teacher knowledge can make a difference in instructional practice and student learning.

Some generalizations regarding *beliefs* have emerged from a synthesis of the existing literature by Ernest (1988), Kagan (1992), Pajares (1992) and Thompson (1992). They include descriptions of beliefs as (a) a personalized form of dynamic knowledge that constrains the teachers' perceptions, judgments and behavior, (b) interpretative filters though which new phenomena are interpreted and meanings ascribed to experiences, and (c) implicit assumptions about content, students and learning. It would appear from these works that beliefs, though different from knowledge, share attributes similar to knowledge. We define beliefs as an integrated system of personalized assumptions about the nature of a subject, its teaching and learning.

In the *Curriculum and Evaluation Standards*, NCTM (1989) has set forth its vision of mathematical power through the articulation of five general *goals* for all students: that they value mathematics, become confident in their ability to do mathematics, become mathematical problem solvers, learn to communicate mathematically, and learn to reason mathematically (p. 5). The NCTM (1991) expects teachers to reflect these goals in their instructional practice. Furthermore, researchers have begun to give increasing attention to goals that emphasize the importance of teaching for conceptual as well as procedural understanding (Cobb et al., 1991; Hiebert, 1986; Silver, 1986). We define *goals* as expectations about the intellectual, social and emotional outcomes for students as a consequence of their classroom experiences.

Comprehensive reviews of research on teacher thought processes have been done by Clark and Peterson (1986) and Shavelson and Stern (1981). Among the components of cognition that seem to impact on instructional practice are *planning* during the preactive stage (Clark & Elmore, 1981; Clark & Yinger, 1979); *monitoring* and *regulating* during the interactive stage (Clark & Peterson, 1981; Fogarty et al., 1983); and *evaluating* and *suggesting* during the postactive stage (Ross, 1989; Simmons, Sparks, Starko, Pasch, Colton & Grinberg, 1989). We share Shavelson's (1986) contention that these aspects of thinking are not conceptually distinct, but rather are interconnected components of a process of developing and implementing agendas based on teaching schemata. From this perspective therefore, we define and categorize these thinking processes as mental activities that teachers use in making decisions and judgments before (*planning*), during (*monitoring* and *regulating*), and after (*evaluating* and *suggesting*) a lesson.

We developed indicators consistent with these definitions of teacher cognitions. We anticipated that data from teacher interviews would provide some evidence of the cognitions underlying the lessons observed. See Table 2 for a list of the specific components of cognition and the indicators.

Insert Table 2 about here

THE STUDY

Subjects

Seven experienced teachers and seven beginning teachers of secondary school mathematics voluntarily participated in this study. Teachers were asked to choose any lesson that would allow for an examination of both their classroom practice and their thoughts underlying that practice. The experienced teachers had taught from seven to twenty-five years. The beginning teachers were student teachers teaching in local middle schools and high schools.

Data Collection

Four types of data were obtained: observation narratives, videotapes of the lessons, audiotapes of the interviews, and lesson plans of the teachers. Transcriptions were made of the audiotapes and the videotapes.

Observations and Videotaping

The first author and a research assistant observed, wrote observation narratives, and videotaped each of the teachers teaching a mathematics lesson of their own design. Observation narratives provided information about classroom occurrences that might not have been visible on the videotape. Transcriptions were made of the audio part of the videotapes for analysis.

Interviews

Immediately following the lesson each teacher engaged in (a) a postlesson structured interview (Interview 1), followed by (b) a stimulated-recall interview as they viewed the videotape of their lesson (Interview 2), followed by (c) a debriefing interview (Interview 3). All interviews were conducted by the first author over a period of one semester.

To determine their cognitions during the preactive stage of the lesson, the teachers were asked in Interview 1 to explain their lesson plans and describe their thoughts as they developed the lesson for the class. They were asked the following questions: (a) Please explain the context in which your plans were made, for example, the type of class, the type of student. (b) What were your areas of concern as you constructed the lesson? (c) What were your main goals for the lesson? (d) What plans or procedures did you intend to use to achieve those goals?

In Interview 2, a stimulated-recall approach was used to determine the teachers' cognitions during the interactive stage of the lesson. As they viewed the videotape of their lesson, they were asked to stop the tape at any point in the lesson where they made a specific decision about what to do next. At each point the tape was stopped, the teachers were asked to describe what they were doing and what they were thinking at that moment.

To determine their postactive cognitions, in Interview 3, the teachers engaged in a debriefing session following their viewing of the videotape. They were asked to reflect on their lessons: (a) Did it go as expected? (b) If they were to teach the lesson again, would they do anything differently? and if so, What?

Note that information regarding the overarching cognitions (knowledge, beliefs, goals) were collected from all three interviews.

Data Analysis

Categorization of Instructional Practice

The instructional practice was coded based on the application of the Phase-Dimension Framework for Assessment of Mathematics Teaching (PDF). The two authors used the observation narratives, the videotapes and the transcriptions of the lessons to rate the dimensions (tasks, learning environment, discourse) of the lessons during the three phases (initiation, development, closure) of instruction. Within each phase of the lesson, each of the five indicators of the three dimensions was rated on a scale of 1 to 3, where a "1" represented an absence of the indicator, a "2" represented a partial presence of the indicator and a "3" represented a consistent presence of the indicator.

To establish reliability of ratings the two authors used five sample videotapes of other mathematics lessons for training purposes. They discussed their interpretations of the dimensions and the indicators and agreed on the criteria for the ratings of "1," "2," and "3." (Descriptions and codings of four lessons will be presented later.) An inter-rater reliability of 91% was established between the two researchers. When there was disagreement they discussed their views and came to a mutual agreement. The five ratings in each category were averaged, and one rating was assigned for each of the three dimensions. Therefore, each lesson received three dimension scores for each of the three phases of the lesson. The average of the nine scores was calculated as was the range of the nine scores. These data were used to categorize lessons on a continuum of lesson quality as *High Quality*, *Medium Quality*, or *Low Quality*.

Categorization of Teacher Cognition

The Teacher Cognitions Framework (TCF) was used to examine teachers' thoughts through an analysis of the interviews and the lesson plans. For each teacher: (a) preactive cognitions (lesson planning) were categorized from the lesson plan and the transcription of Interview 1; (b) interactive cognitions (monitoring and regulating) were categorized from the transcriptions of Interview 2; and (c) postactive cognitions (evaluating and suggesting) were categorized from the transcriptions of Interview 3. Note that the overarching cognitions (knowledge, beliefs, goals) were categorized from the lesson plans and from the transcriptions of all three interviews. A descriptive analysis was given in terms of the indicators for each component of cognition.

Descriptive Analysis of Teacher Cognitions Underlying Instructional Practice

For each category of lesson quality, determined by the PDF, we examined the data from the TCF to see if any patterns emerged. We then described the patterns of cognitions associated with each category of lesson.

RESULTS AND ANALYSIS

Analysis of Teacher Cognition in Relation to Instructional Practice

Using the PDF we were able to categorize the lessons. The average scores of the three dimensions across three phases of each lesson ranged from a low of 1.24 to the highest possible score of 3.00 and the range of each lesson's scores varied from .00 to 2.00. The data yielded information about the consistency or lack of consistency of lesson quality over the course of a lesson. The lessons that received high ratings and the lessons that received low ratings both had a lower range of scores than the lessons in the middle category indicating a greater consistency of lesson quality. On the basis of these findings the lessons were divided into three categories of lesson quality: *high quality, medium quality*, and *low quality*. That is, lessons with high ratings (2.53 to 3.00) and a low range of individual scores (.00 to 1.00) were rated as *high quality,* lessons with low ratings (1.23 to 2.20) and a low range of scores (1.00 to 1.40) were rated as *low quality,* and lessons with middle range ratings (2.20 to 2.40) and a high range of scores (1.20 to 2.00) were rated as *medium quality*. For ease of discussion the lessons coded as *high quality, medium quality* and *low quality* will be referred to as Group HQ, Group MQ, and Group LQ respectively.

By applying the Teacher Cognitions Framework (TCF) to the interview data and the lesson plans, we were able to describe the teachers' cognitions during three stages of instruction: preactive, interactive and postactive. Patterns of cognitions were found when the TCF results were organized according to the PDF ratings of lesson quality.

The results are presented according to categories of lesson quality. That is, for each category, a description of the characteristics of the lessons and the related cognitions underlying these lessons are given. Woven through these descriptions will be an example of one lesson from the category and the related cognitions of the teacher of that lesson. To most clearly exemplify the ideas, the most extreme cases were chosen for highlighting. For example, the lesson highlighted in the HQ category was the lesson that had the highest quality rating. (See the Appendix A for the instructional practice ratings of all highlighted lessons.)

Instructional Practice and Underlying Cognitions of Teachers of High Quality Lessons (Group HQ)

Five lessons were coded as *high quality* (Group HQ). Four were taught by experienced teachers and one was taught by a beginning teacher. The five lessons had the following total averages and range of scores (in parentheses): 3.00(0.0), 2.91(0.8), 2.89(0.6), 2.76(0.8), 2.53(1.0). As these numerical results indicate, a fairly consistent pattern of high quality instruction was noticed for the dimensions of tasks, learning environment, and discourse throughout all phases of their lessons. A descriptive analysis revealed a similar consistency in the focus of the expressed cognitions of these teachers. That is, their knowledge, beliefs and goals centered around student learning with understanding, as did their thought processes before, during and after the lesson.

A more detailed description of the results follows with specific examples taken from Gina's lesson and cognitions. Gina's lesson received the highest rating of lesson quality. She was in her tenth year of teaching high school mathematics and was observed teaching a geometry lesson on proving overlapping triangles congruent to a class of 33 tenth graders in an urban high school.

Preactive. In their preactive interviews the Group HQ teachers revealed goals for their students to attain both procedural and conceptual understanding of the content. They also wanted their students to see the value in the mathematics they were learning. They showed knowledge of the content, pedagogical techniques and students in that they were able to: 1) describe the content in relation to the students' past and future study; 2) describe the difficulties they anticipated in students' learning of the content; and 3) describe suitable pedagogical strategies they planned to use.

In her preactive interview Gina stated that, more than just having students learn how to solve overlapping triangle problems, her primary goal was that they understand the methodology involved. She wanted them to learn the problem-solving strategies appropriate for such problems: using different colored pencils or redrawing the diagram as a means for better visualization of the parts of each triangle. She gave a detailed explanation of the students' readiness level for this topic and purposely designed an introductory problem that would link with their previous knowledge. Anticipating their difficulty with writing proofs, she decided to have them write a plan for the first proof before having them actually do it. Anticipating their difficulty with visualizing the overlapping triangles, she was prepared with colored chalk to aid in clarification.

Interactive. The instructional practice of the Group HQ teachers was consistent with their preactive cognitions which revealed a concern for and knowledge about students, content and pedagogy. That is, the tasks appeared to be interesting to students, logically sequenced, and at a suitable level of difficulty. An appropriate use of tools and organization of tasks contributed towards the clarity of the lesson. There was a relaxed, yet businesslike learning environment in which most of the students appeared to be on task. The instructional routines and pacing were appropriate for active student involvement in the lesson. The administrative routines were handled effectively and efficiently.

During their stimulated-recall interviews, these teachers made specific comments regarding their beliefs about the necessity of a student-centered approach for student learning. Furthermore, they gave descriptions of how they used student participation and feedback as a means of monitoring student understanding, which they used for subsequent regulation of instruction. The discourse during the instructional practice was consistent with these cognitions. That is, the teachers encouraged the students to think and reason, give full explanations of their thoughts, and listen to and respond to one another's ideas. This type of discourse seemed to have been facilitated through the teachers' use of a variety of types and levels of questions with appropriate wait times.

Gina's emphasis on having her students engage in reasoning emerged as soon as her students entered the class and were presented with a geometric proof. Gina told them that rather than doing the proof, she wanted them only to write out a plan for how they would do the proof. In her interview she said, "I wanted to focus in on a plan that we needed to have. Lots of times in the past, they don't know what they're doing. They don't know how to think about it, where they're going. So they have to have a definite plan." After about five minutes, she brought the class together and engaged them in further discussion that focused on the analysis of the problem. Through questioning she got the students to link their prior understandings with new ideas by asking questions that got them to focus on what was different about the proof they were being asked to do than the ones they had done in previous lessons. After realizing that the triangles they were being asked to prove congruent had overlapping parts, she asked how they might simplify this situation to make this problem look like ones they had done before. She waited until students suggested such ideas as using colored chalk to highlight the different triangles or even redrawing the triangles by translating one so that it would be separate from the other. Upon completion of the proof, she asked the students to look back on the work to review unusual aspects of the problem and to explain the approach they used to solve it.

Gina's careful sequencing was evidenced as she engaged the students in two more proofs of increasing difficulty. One was reviewed with teacher guidance and the other was done as seatwork while the teacher walked around monitoring student work and providing them with feedback. All diagrams were drawn with rulers and the boardwork was accurate and well organized. At the end of the lesson the teacher engaged the students in a summary of the work they had done and the strategies they had used. For reinforcement she gave them diagrams in which their only task was to locate the overlapping parts of the triangles.

Throughout this class problem-solving session Gina demonstrated the following questioning behaviors: (a) she waited until a number of hands were raised before selecting a student to answer a question, (b) she encouraged students to comment on each other's ideas and to direct their questions to other students rather than her, (c) students who spoke too quietly were asked to repeat themselves, and (d) when students gave incorrect responses she did not pass judgment but rather encouraged other students to consider the question. As she watched her questioning techniques on the videotape she spoke about how she liked to give all students the chance to think She said, "There are a few good students who always have their hands up. They would, if I let them, dominate the class and nobody else would ever have time to think." She explained that she let other students respond to student questions to keep them on their toes and also because she recognized that one student's question or error is probably indicative of many other students' questions or errors. She spoke about how she "...would like (students) to talk more, listen to each other, because that's how you learn." She spoke about changes she made in her lesson plan based on the feedback she was getting from students and based on the pacing and flow of the lesson.

__Postactive__. In their debriefing interviews the Group HQ teachers showed a consistency with their preactive goals, in that they rated their lessons primarily in terms of their evaluation of how much their students understood. Finally, they gave detailed suggestions for improving their instructional techniques aimed at increasing clarity and interest for students.

Gina demonstrated her priority for student understanding over content coverage at the completion of her lesson. That is, she expressed satisfaction with her lesson, even though she had not completed all that she had planned. She was pleased to note all the hands that were raised and the level of understanding she thought they had achieved. She gave suitable suggestions for making the material clearer by using an overhead transparency or cut outs to actually separate overlapping triangles in a figure.

Instructional Practice and Underlying Cognitions of Teachers of Low Quality Lessons (Group LQ)

Four of the lessons were coded as *low quality* (Group LQ). All four were taught by beginning teachers. The four lessons had the following total averages and range of scores (in parentheses): 2.20 (1.0), 2.07(1.2), 1.78(1.4), 1.24(1.2). As the numerical results indicate, a fairly consistent pattern of low quality was noticed for the dimensions of tasks, learning environment, and discourse throughout all phases of the lessons. A descriptive analysis revealed that the expressed cognitions of these teachers were consistently focused on their own practices rather than on student learning. That is, their knowledge, beliefs and goals centered around content coverage for skill development and management concerns, as were their thought processes before, during and after the lesson.

A more detailed description of the results follows with specific examples taken from Ellen's lesson and cognitions. Ellen's lesson received the lowest rating of lesson quality. Ellen was a student teacher in an urban middle school. She was observed teaching a first lesson on graphing linear equations to a class of 26 seventh graders.

<u>Preactive</u>. In their preactive interviews the Group LQ teachers expressed only procedural goals for their students and desires to cover the content. None made any mention of any conceptual or affective goals for the students. They revealed only a general and vague knowledge of their students, the mathematical content, and related pedagogy. They spoke about the content in isolation and focused mainly on time-saving management strategies to cover the content.

In her preactive interview, Ellen stated only the procedural goal that her students "...learn how to plot a line." When asked how she intended to accomplish the goal she gave the noncontent specific comment, "I wanted to show the difference between what we solved before and what we're doing. I just wanted to do one example and to just get right into the graphing because that's how they learn." She expressed a general concern that some students might get bored since she suspected that some of the students already knew the material. However, she did not specify the aspects of the material she thought they knew.

<u>Interactive</u>. The instructional practice of the Group LQ teachers was consistent with their cognitions in that the tasks were illogically sequenced, and either too easy or too difficult for the students. Both the poor organization of the content and the inappropriate use of tools masked the clarity of the concepts. Ineffective and inefficient instructional routines, combined with inappropriate pacing, contributed towards a tense and awkward classroom atmosphere in which many of the students appeared to be off task. The administrative routines were accomplished but in a way that was time consuming and/or disruptive to the lesson.

The verbal interactions during the lesson reflected these teachers' cognitions. The absence of monitoring for student understanding during the lessons was evidenced in the nature of the discourse. The teachers asked low-level, leading questions with inappropriate wait times. They did not require students to give explanations of their responses, nor did they encourage interactions between and among students. They passed judgment on student responses and often resolved questions without student input.

During the stimulated-recall interviews, unlike the Group HQ teachers, the Group LQ teachers made no statements regarding their beliefs about how students learn best. They gave descriptions of how they monitored student behaviors as a means for improving classroom management but made no mention of monitoring for student understanding. In fact, none of these teachers described or made any deviations from their original plans, despite feedback from students during the course of the lesson that indicated they were confused.

The weak learning environment in Ellen's class was first evidenced as the students entered her class in a disorderly and noisy fashion. Ellen yelled over the noise to give instructions and chatted with individual students as they entered the room. Although there were three problems on the board for students to do, none of the students appeared to be doing them. In her interview she explained that the one reason she couldn't get their attention was because they were being filmed. Ellen told the students to take out their homework, which involved the task of plotting fourteen pairs of points on a coordinate grid. While the class was still noisy she placed a grid, that was hard to read, on the overhead projector and called on students, who were admittedly unprepared, to come up to the overhead and plot the points. Since the students were unprepared, this was a time-consuming instructional routine. In her interview, she revealed that her reasons for calling on students who did not do their homework were management related only: "...it wasn't that difficult to do and they could have done it right away and it's a way of me knowing, and the whole class knowing, that they did not do it."

Throughout the lesson, Ellen asked all low-level questions, accepted responses from students who yelled out, and passed judgment on the responses. When a response was incorrect, she made such derogatory comments as "How could you forget that?" and then engaged in a one on one conversation with the student, further losing the class' attention. She expressed bewilderment about why the students gave incorrect responses and explained it by saying, "I think he wasn't thinking," "They don't think." She then switched to the work from the Do Now which involved solving equations in one variable, which was unrelated to the work done for homework. In another awkward transition, she stated that they would now do something different. She then asked a vague question, followed by a confusing nonexample to try to introduce the idea of solving equations in two variables. That is, she misled students to believe that $x + y = 6$ was different from $x + x = 6$, since x and y had to be different. The students came to assume that x could not equal y. While viewing the confusion of her students, she said, "...It's just hard to explain and it's just hard for them." Finally, she was able to elicit pairs of solutions for the equation, which she recorded in a disorganized fashion on the board. She made another awkward transition that was caused by inappropriate sequencing when she forced students to redo their work by calculating the same pairs of values, only this time listing them in a table. In her interview, she explained that she forced them into doing the table since that's what they would be required to do on the homework the following day. The next awkward transition occurred as the teacher announced, "While you finish that table, we are going to try to graph your ordered pairs." She then graphed the equation herself without the attention of the students. Some people were trying to graph the points on their own but were having difficulty. The teacher said she would help them, but never did. She kept asking students if they saw a pattern (that is, that the points formed a line). But, by their lack of responses it appeared that the students didn't seem to know what she was talking about. She expressed bewilderment about the low level of participation and the fact that the students didn't understand the work. She said, "It just never occurred to me that that would be a problem." As time was running out, she tried to draw closure to the lesson by having them "make up some rules on how to graph." Having graphed only one equation, this was inappropriate. Furthermore, when she gave the rules, they were not in line with the procedure they had actually followed. The students left the class while the teacher was still speaking about the content of the homework and what they would be doing in class the following day.

Postactive. In their postactive interviews, the Group LQ teachers showed a consistency with their preactive goals in that their primary focus was on their insufficient content coverage and the student behavior. Several gave suggestions for improvement of the pacing of their lessons to achieve more efficient content coverage.

In her postactive interview Ellen's only stated concern was her insufficient content coverage. She said that she "...didn't get to cover as much as [she] thought. I thought it would go quicker. There wasn't enough time to summarize, to put it all together at the end." When asked whether she would do things differently if she were to teach the lesson again, she gave a suggestion for saving time. That is, she spoke about not requiring the students to organize the data pairs in a table. When asked how she felt about the lesson, she said, "It was good. I think the kids were very good today. Some kids were still not paying as much attention as I'd like."

Instructional Practice and Underlying Cognitions of Teachers of Medium Quality Lessons (Group MQ)

Five lessons were coded as *medium quality* (Group MQ). Three were taught by experienced teachers and two were taught by beginning teachers. These lessons were rated as *medium quality*, since each had components that resembled those of both *high quality* lessons and *low quality* lessons. On their ratings of instructional practice, the five lessons had the following total averages and range of scores (in parentheses): 2 40(2.0), 2.40(2.0), 2.37(1.2), 2.20(1.8), 2.20(1.8). As the relatively large ranges indicate, a fairly inconsistent pattern of lesson quality was noticed. A descriptive analysis revealed a similar inconsistency in the focus of the expressed cognitions of these teachers. In some essential characteristics, to be explained subsequently, the lessons and cognitions of three of the teachers, two experienced and one beginning (to be referred to as Group MQ1) were similar, and the lessons and cognitions of two of the teachers, one experienced and one beginning (to be referred to as Group MQ2) were similar.

Medium Quality Lessons (Group MQ1)

The similarity of these Group MQ1 lessons was that they each were rated high on *tasks* and *learning environment*, but low on *discourse*. More specifically, a teacher dominated style of discourse predominated in which little monitoring for student understanding occurred during any phase of the lessons. Similarity among the beliefs of these teachers shed light on their instructional practice.

A more detailed description of the results follows with specific examples taken from Betty's lesson and cognitions. Betty was in her seventeenth year of teaching secondary school mathematics. She was observed teaching a geometry lesson on proving the properties of isosceles triangles to a class of 22 tenth graders in a suburban high school.

Preactive. In their preactive interviews the three Group MQ1 teachers revealed goals for their students similar to those of the Group HQ teachers. That is, they wanted them to develop conceptual as well as procedural understanding of the content. The teachers exhibited detailed knowledge about their pupils, the content and related pedagogy.

In her preactive interview, Betty stated both procedural and conceptual goals to "reinforce previous concepts" and to get the students to "...realize what does happen in an isosceles triangle." She revealed knowledge and consideration of the ability level of her students. She stressed the importance of reinforcing previous concepts and proper sequencing of problems. Her statement, "I know the speed at which to go with this class," showed that pacing was a priority for her. She said that, in fact, she was not in a rush since "...we're right on target with time."

Interactive. With respect to the *tasks* and most aspects of the *learning environments*, the instructional practice of each of the three Group MQ1 teachers resembled the instructional practice of the Group HQ teachers. That is, throughout all lesson phases the tasks were logically sequenced and at a suitable level of difficulty. An appropriate use of tools and organization of tasks contributed towards the clarity of the lessons. The mathematical content appeared to be moderately interesting for students. The administrative routines were handled effectively and efficiently and there was a relaxed, yet businesslike learning environment in which most of the students were on task.

With respect to the *discourse*, however, the instructional practice of the Group MQ1 teachers resembled that of the Group LQ teachers. That is, the pacing and instructional routines were not conducive to student input. In addition, the discourse was largely teacher centered with fast-paced (short wait time), low-level teacher questions requiring one word answers from students. Students were rarely encouraged to interact with one another and the teachers often passed judgment on student responses and resolved questions without student input. Despite indications from students that they were confused, each of these teachers taught their lesson without deviation from their original plans.

During their stimulated-recall interviews, the Group MQ1 teachers revealed beliefs quite different from those expressed by the Group HQ teachers, who valued the student-centered approach to teaching. Both of the experienced Group MQ1 teachers stated that in order to cover the content efficiently, a more teacher-centered approach would be desirable. That is, they believed it was best to tell students the information rather than spend the time getting them to discover it for themselves. The beginning teacher expressed her uncertainty regarding how much to elicit from the students and how much to tell them, in light of the time constraints of a class period and her concerns about covering the content. Similar to the Group LQ teachers, as they viewed their lessons, all three teachers explained that the primary reason they called on students was to keep them on task.

The positive qualities of the learning environment and tasks in Betty's class became apparent as the students entered the class and were met by Betty, who immediately presented them with a worksheet. The students cooperated by sitting immediately, taking out their homework and placing it at the corner of the desk for the teacher to check, and beginning their work on the worksheet. In her interview Betty stated that, "It's very important to me that at the beginning of each period they settle in quickly. The Do Now (the worksheet) serves the purpose for me of reviewing constantly and, when I can, leading into the lesson of the day, using parts of it." During this initial phase of the lesson, a student asked if he could go to his locker to get his homework. Betty did not allow this and told the class that they were expected to come to class prepared. While the students did their work, Betty checked attendance, walked around checking homework, and also checked the work they were doing in order to select students to put their work on the board. She stated that her purpose for checking the students' homework at their desks was to ensure they attended to their tasks: "...they know that there's accountability, that they have to do it." Her mathematical tasks were carefully prepared and suitably sequenced for the clear development of concepts.

Like the other Group MQ1 teachers, the quality of the discourse in Betty's class did not measure up to the quality of the learning environment in her class or the qualit of the tasks she designed. Specifically, after a student put his or her work on the board, Betty's overriding questioning technique was to ask the student an easy procedural question, allow a very short wait time, accept a one-word answer from the student, and then give the explanation for the student answer herself. For example, the following verbal interaction took place regarding one proof:

Betty: "OK, now, I'm interested in number 3. AD is perpendicular to BC. Sam, you said that AD is an altitude. Why?"

Sam: "Because, right angles."

Betty: "Um huh. An altitude is a line segment that goes from the vertex and is perpendicular to the opposite side. OK, and AE, I said is the middle. Why is it a median? What is the definition of a median? Scott?"

Sam: "It makes BE = EC."

Betty: "Well, a median is a line segment which goes from the vertex of the triangle to the middle of the opposite side."

As Betty watched her lesson on the videotape, she said, "Sometimes I let students explain their work. But because of time factors, I took charge.... It works well when the teacher stands in front of the room and answers student questions." This view seemed inconsistent with her preactive assertion that time was not a problem.

Postactive. Similar to the Group LQ teachers, in their debriefing interviews, the Group MQ1 teachers evaluated their lessons in terms of content coverage and gave suggestions for improvement that focused on ways to accomplish more efficient pacing. This was inconsistent with their preactive cognitions which, similar to the Group HQ teachers, focused on helping students to attain procedural as well as conceptual understanding.

During her postactive interview, similar to the other Group MQ1 teachers, Betty's evaluation was focused predominantly on content coverage: "I accomplished what I wanted to."

Medium Quality Lessons (Group MQ2)

The similarity of the two Group MQ2 lessons was that they each were rated high on all dimensions (*tasks, learning environment, discourse*) during the **initiation** phase of their lessons. However, they were rated low on *tasks* and *discourse* during the **development** and **closure** phases of their lessons. Similarity in the extent of the knowledge they revealed shed light on this instructional practice.

A more detailed description of the results follows with specific examples taken from John's lesson and cognitions. John was a student teacher in an urban middle school. He was observed teaching a lesson on plotting points on a rectangular coordinate system to a class of 30 seventh grade students.

Preactive. Similar to the Group HQ teachers, in their preactive interviews the two teachers in this group revealed goals for their students to develop conceptual as well as procedural understanding of the content. They also expressed beliefs about the importance of having students play an active role in their own learning by asking them questions and challenging them to think for themselves and interact with one another. However, unlike the Group HQ teachers, these teachers either admitted to or demonstrated that they had inadequate or superficial knowledge about some aspects of the content, students, and/or pedagogy.

In his preactive thoughts, John stated procedural and conceptual goals. He wanted his students to learn how to graph a point and, at the same time, review the geometric concepts and enable them to make the relation between the two. However, like the Group LQ teachers, he revealed only a general and vague knowledge of his students by anticipating that "it would be an easy lesson and [he] wouldn't have any difficulty." To accomplish his goals, he said, "I plan to help them to do it instead of to do it myself. I'm going to send them to the board."

Interactive. During the **initiation** phase of their lessons the *tasks*, *learning environment*, and the *discourse* resembled those of the Group HQ lessons. That is, the tasks were logically sequenced and at a suitable level of difficulty, the discourse was student-centered and there was a relaxed, yet businesslike classroom climate. However, during the **development** and **closure** phases of their lessons the *tasks*, and *discourse* resembled those of the Group LQ lessons. That is, the tasks were either too difficult or confusing for students, the discourse was teacher centered and the instructional routines and pacing were teacher dominated and not conducive to student input.

As they watched the videotape of the initiation phase of their lessons, similar to the Group HQ teachers, the Group MQ2 teachers claimed that they called on students to check for understanding so as to know how to proceed. However, as they watched the development and closure phases of their lessons, they remarked that the tasks they introduced caused confusion which required them to tell the students the information.

As John's class began, students received worksheets that concerned the plotting of points on a graph, which they worked on individually at their seats. John circulated around the room checking this work. He said, "I was trying to see if they are able to do the Do Now, cause if they can't do it, forget it. And I saw that some of them had some difficulty." After all were finished, one student at a time was selected to put his or her work on the board and explain it to the class. The work consisted of plotting one point on the graph. When the students were at the board, the teacher encouraged the seated students to question them about their work. He allowed long wait times and placed the burden of evaluation on the students. While watching the tape of his lesson, he expressed his beliefs about the value of student input: "If a kid can do it, I prefer if he explains. It helps him." He acknowledged that he does this, in spite of the fact that it takes more time from the class than if he would just explain it himself.

During the development phase of the lesson, John assigned a complex problem, where the students had to plot four points, join the points, and find the perimeter and area of the resulting figure. The students showed a lack of familiarity with the concepts of perimeter and area. Therefore, the teacher was unable to elicit the responses he wanted in the short remaining time. He thus resorted to telling the students how to do the problem and gave them the answers as the bell rang. As he watched this phase of his lesson on the videotape, he said, "I was thinking how long this part is going to take me, but I have to do it." He explained that he made a choice to forego his prepared summary and give this complex problem, since it resembled that evening's homework assignment.

Postactive. In their debriefing interviews, similar to the Group HQ teachers, both teachers evaluated student understanding and gave appropriate suggestions for how to improve the design of the tasks in their lessons. Both teachers claimed that their inadequate knowledge of the content, students, and/or pedagogy impeded their efforts to teach in a way that was consistent with their goals and beliefs.

During his postactive interview, John recognized that he belabored the point plotting and he noted that his last example was inappropriate: "I was too ambitious. I'm not sure that many knew what was going on. I should have just focused on plotting points and not include area and perimeter."

To summarize these results we have organized the data in two charts: one that summarizes the patterns of lesson dimensions and one that summarizes the patterns of cognitions. See Appendixes B and C respectively.

Through the application of our model we were able to discern that teacher cognitions played a well-defined role in instructional practice. That is, patterns of cognitions paralleled differences in the quality of instructional practice.

DISCUSSION AND IMPLICATIONS

The purpose of this study was to examine the cognitions associated with lessons of differential quality. We used the Phase-Dimension Framework to differentiate fourteen mathematics lessons. We then used the Teacher Cognitions Framework to systematically describe the cognitions of the teachers of these lessons.

Using the Phase-Dimension Framework, lessons were rated as either *high quality, medium quality*, or *low quality* based on the level and consistency of ratings in the dimensions of teaching (tasks, learning environment, discourse) across the three phases of instruction (initiation, development, closure). Lessons rated as *high quality* (four taught by experienced teachers, one taught by a beginner) were consistently high on the dimensions within each phase of the lesson. Lessons rated as *low quality* (all four taught by beginners) were consistently low on the dimensions within each phase of the lesson. Lessons rated as *medium quality* (three taught by experienced teachers, two taught by beginners) received inconsistent ratings. Some received high ratings on the dimensions of task and learning environment, but low ratings on discourse, throughout each phase of the lesson. Others received high ratings on all three dimensions during the initiation phase of their lessons, but low ratings on task and discourse during the development and closure phases.

The findings from the application of the Teacher Cognitions Framework provided some insights for the variations in the instructional quality of these lessons. The verbal data of the teachers of lessons rated as *high quality*, revealed that their knowledge, beliefs and goals were focused on student learning with understanding, as were their thought processes before, during and after the lesson. Apparently, when issues about learning and how to promote it among students are central in teachers' cognitions, teachers' observable behaviors in the classroom correspondingly demonstrate this focus. Specifically, teachers having these cognitions demonstrated instructional practice that was characterized by well-designed tasks and intellectually and socially stimulating learning environments where the discourse fosters interaction in which students share responsibility for their own learning. The teachers' extensive monitoring of this rich verbal interaction may have accounted for their subsequent accurate postlesson judgments regarding whether they had accomplished their goals of teaching for student understanding. The monitoring behaviors these teachers demonstrated were similar to those of expert teachers (Borko & Shavelson, 1990; Livingston & Borko, 1990; Leinhardt & Greeno, 1986) and good problem solvers (Schoenfeld, 1987; Silver, 1985). Somewhat surprising, though, was the finding that these competencies, usually associated with expertise were within the repertoire of the skills of a beginning teacher. This has positive implications for preservice teacher trainers as well as school-based professionals who employ beginning teachers. That is, although experience plays an important role in the development of a teacher, it is certainly possible for a beginning teacher to think and teach in ways similar to experienced teachers of high quality lessons.

In contrast, the verbal data of the teachers of lessons rated as *low quality* revealed that their knowledge was fragmented, goals were limited to isolated performance outcomes for students, and no overarching beliefs were articulated. Moreover, the teachers' thoughts before, during and after the lesson revealed minimal attention to students' learning. Specifically, it appeared that when there was deficient knowledge and an absence of goals for student understanding it was accompanied by poorly designed tasks, ineffective learning environments and the absence of verbal interaction that reflected monitoring for student understanding. This lack of monitoring may have accounted for their subsequent inaccurate postlesson judgments that their lesson went well or that their students understood. In some ways, the four beginning teachers of these lessons exhibited behaviors similar to those of other novice teachers (Borko & Livingston, 1989; Livingston & Borko, 1990) and naive problem solvers (Hinsley, Hayes & Simon, 1977). These findings have important implications for preservice teacher educators whose primary goal is to empower teachers to teach for student understanding and to reflect on their practice as a means for self improvement. Teachers must come to believe that their primary goal of instruction should be student understanding, and that the only way to accurately gauge how well they are accomplishing this goal is to actively monitor student understanding through appropriate discourse during instruction.

The teachers of the lessons rated as ***medium quality*** also revealed a lack of consistency within or among components of their cognitions. For some lessons, the teachers expressed knowledge and goals consistent with teaching for the promotion of student understanding. However, their lack of monitoring for student understanding, during the interactive stage of teaching, was inconsistent with these preactive cognitions. Furthermore, they showed an unawareness of the lack of coherence between their postlesson cognitions, which focused only on content coverage and more efficient pacing in subsequent lessons, and their preactive cognitions which focused on student understanding. For these lessons, teachers' beliefs about the value of "teacher telling" may have accounted for the persistent use of teacher-dominated strategies for discourse, which resulted in the absence of monitoring for student understanding across all phases of their lessons. During their postactive interviews, they expressed the belief that when time is at a premium, covering the content efficiently must take precedence over student learning with understanding. Like the teachers in Lampert's work (1985), but unlike those whose lessons were rated as *high quality*, these teachers were unable to maintain the "tension" between simultaneously covering the content and attending to student understanding. This finding implies that if facilitation of student understanding of mathematics is an important aspect of quality instructional practice, as the literature suggests, then we must find ways to enable teachers to cope effectively with this paradox of teaching.

For other lessons rated as *medium quality*, the teachers revealed beliefs and goals that suggested the importance of student learning with understanding. However, because of their inadequate or superficial knowledge about the content, students and/or pedagogy, in the development and closure phases of their lessons, they were unable to monitor and regulate their classroom teaching in a manner consistent with their preactive cognitions. For teachers of these lessons, regardless of experience, weaknesses in different aspects of their knowledge was the major source of difficulty - a problem that diminishes the quality of teaching (cf., Peterson, 1988; Shulman, 1986b). Specifically, after the initial phase of their lessons, when they realized that they had (a) introduced tasks that were causing confusion for themselves or for their students, and (b) did not know how to adjust the tasks, they resorted to a teacher-centered lesson. One might say that a teacher-directed style of teaching can serve as a mask for teachers who do not possess full knowledge of the content, students and/or pedagogy. That is, without the demands arising from student input, teachers are free to impose the material on the students even when they themselves do not fully understand it or have inappropriately sequenced the material. With the present emphasis on the critical role of discourse for the teaching of mathematics with understanding, teacher education programs must provide opportunities for preservice and inservice teachers to develop a thorough and integrated knowledge system.

The absence of monitoring for student understanding was a common weakness in lessons rated as *low quality* and in all or parts of the lessons rated as *medium quality*. Equally troubling, though, was teachers' apparent unawareness of the importance of monitoring for student understanding as a means towards accurate postlesson judgment of student understanding. This has important implications for teacher educators since, accurate postlesson assessment of student understanding is an important means of obtaining more information about students for subsequent planning and classroom practice. Preservice and inservice teachers must become aware of the centrality of the role of monitoring and regulation of student understanding for both effective teaching of mathematics and continued professional development. This notion is consistent with the research on problem solving which shows that cognitive monitoring and subsequent regulation play a pivotal role, not only in the efficacy of the problem solving process, but in the ultimate solution of the problem. (Artzt & Armour-Thomas, 1992; Garofalo & Lester, 1985; Schoenfeld, 1987).

Despite the promising nature of these results, there are some limitations to this exploratory study. First, there was no formal assessment of student learning in the sample of lessons observed. Observations of the teaching-learning transaction should be used in conjunction with other procedures to ascertain what and how much students have learned from their classroom experiences. Secondly, a larger number of observations of lessons and interviews would contribute to greater validity of the findings of both frameworks. Finally, although the Teacher Cognitions Framework yielded valuable information on the thoughts of teachers, they were derived only from the comments that the teachers volunteered. These results would need to be complemented with other data sources to tap teacher cognitions, such as questionnaires or experimental tasks as well as data indicating student understanding.

Through the application of two frameworks, we were able to examine the teaching of mathematics as an integrated whole and obtain a better understanding of instructional practice and associated teacher cognitions. Specifically, we used the Phase Dimension Framework for the Assessment of Mathematics Teaching to differentiate the lesson quality of fourteen mathematics lessons. We then used the Teacher Cognitions Framework which revealed patterns of teachers' cognitions associated with these lessons. With further refinement, these frameworks may prove useful to researchers and teacher educators in their preservice and inservice mathematics programs. They may now approach teaching as an integrated whole, where cognitions play a well-defined role in instruction.

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Appendix A

Instructional Practice Ratings of the Four Highlighted Lessons

Teachers		**Gina**			**Ellen**			**Betty**			**John**		
Lesson Phases		INIT	DEV	CLO	INIT	DEV	CLO	INIT	DEV	CLO	INIT	DEV	CLO
DIMENSIONS	INDICATORS												
Tasks	Tools/Materials	3	3	3	2	2	1	3	2	3	3	3	3
	Motivational Strategies	3	3	3	2	1	1	3	2	2	2	2	2
	Content	3	3	3	2	2	1	3	3	3	3	3	1
	Difficulty Level	3	3	3	3	2	1	3	3	3	3	3	1
	Sequencing	3	3	3	2	1	1	3	3	3	3	3	1
Learning Environment	Social/ Intellectual Climate	3	3	3	2	2	1	3	3	3	3	3	3
	Administrative Routines	3	3	3	1	1	1	3	3	3	3	3	3
	Instructional Routines	3	3	3	1	1	1	3	3	3	1	1	1
	Pacing	3	3	3	1	1	1	2	2	2	1	1	1
	Student Deportment	3	3	3	1	1	1	3	3	3	3	3	3
Discourse	Questioning	3	3	3	1	1	1	1	2	2	2	2	1
	Student Responses	3	3	3	1	1	1	1	2	2	3	3	1
	Teacher-Student Interactions	3	3	3	1	1	1	1	2	2	3	3	1
	Teacher Responses	3	3	3	1	1	1	1	2	2	2	2	1
	Student-Student Interactions	3	3	3	1	1	1	1	2	1	3	2	1

3 = Presence of indicator
2 = Partial presence of indicator
1 = Absence of indicator

Appendix B

Summary of Patterns of Lesson Dimensions

Lesson Dimensions	Dimension Indicators	High Quality Group HQ	Middle Quality Group MQ1			Middle Quality Group MQ2		Low Quality Group LQ
Tasks	Tools/ Materials	Appropriate and accurate use of tools and or materials to aid in clarity in an efficient manner	H	H	H	B	H	Ineffective and/or inappropriate use of tools and or materials to aid in clarity of lesson
	Motivation	Relevant, interesting tasks integrated throughout lesson	B	B	B	B	B	Tasks unrelated to student interests
	Content	Organization of tasks appropriate to clarify concepts	H	H	H	B	B	Inappropriate organization of tasks for clarifying concepts
	Difficulty Level	Tasks challenging yet within reach of students' abilities	H	H	H	B	B	Tasks often too easy or too difficult for students
	Sequencing	Tasks logically sequenced	H	B	H	B	B	Tasks illogically sequenced
Learning Environment	Social/ Intellectual Climate	Relaxed, yet businesslike atmosphere. Lesson centered around student input	H	H	B	H	H	Tense, awkward atmosphere. Superficial requests for and use of student input
	Administrative Routines	Effective and efficient routines	H	H	H	H	H	Effective but not efficient
	Instructional Routines	Appropriate arrangements for student participation and efficient goal attainment	H	B	H	H	L	Ineffective and inefficient procedures for student participation and goal attainment
	Pacing	Pacing appropriate for student involvement and exploration of ideas	B	B	L	B	L	Pacing either too fast to involve students or too slow to maintain interest
	Student Deportment	Students generally on task	H	H	H	H	H	Students often off task
Discourse	Questioning	Variety of levels and types of questions. Appropriate wait times	B	B	L	B	L	Low-level, leading questions. Inappropriate wait times
	Student Responses	Students give explanations and justifications of responses	B	B	L	B	B	Students do not give explanations or justifications for their answers
	Teacher-Student Interactions	Teacher shows accepting attitude toward students' ideas and invites other students' input	B	B	L	B	B	Teacher passes judgment on students' responses
	Teacher Responses	Teacher responds in variety of ways to encourage students to think and reason	B	B	B	B	L	Teacher resolves questions without student input
	Student-Student Interaction	Students listen to and respond to each other's ideas and questions	L	L	L	B	B	No interaction between and among students

H = Cognitions resembled those of teachers in Group HQ
L = Cognitions resembled those of teachers in Group LQ
B = Cognitions resembled those of teachers in both groups.

Appendix C

Summary of Patterns of Cognitions

Cognitions	Components	High Quality Group HQ	Middle Quality Group MQ1			Group MQ2		Low Quality Group LQ
Overarching	Knowledge Pupils	Specific knowledge of students' prior knowledge, abilities and attitudes	H	L	H	H	L	General and superficial knowledge of students
	Knowledge Content	Viewed content in relation to entire unit and past and future study	H	L	H	L	L	Content viewed in isolation of past and future study
	Knowledge Pedagogy	Anticipated specific areas of difficulty and planned suitable teaching strategies	H	H	H	H	L	Primary focus on time saving management strategies to cover the content
	Beliefs Student Role	To think, discover, communicate, and take responsibility for learning	L	B	L	H	H	To stay on task
	Beliefs Teacher Role	To ask questions that challenge students to think for themselves and interact with one another	L	B	L	H	H	To model how to do problems
	Goals	Primary focus on conceptual understanding, procedural skills and appreciation of content	B	H	B	H	L	Primary focus on content coverage and students' procedural skills
Preactive	Lesson Planning: Objectives	Focus on problem-solving processes and conceptual meanings and underlying procedures and results	L	L	L	B	B	Focus on procedures and results
	Lesson Planning: Structure	Problems logically sequenced from easy to more difficult. Ideas built on students' past knowledge	H	H	H	B	B	Problems illogically sequenced. Large leaps in concepts and confusing examples
	Lesson Planning: Phases	Appropriate integration of initiation, development and closure	H	H	H	B	B	Content within phases inappropriate in relation to other phases
Interactive	Monitoring	Called on students to increase participation, evaluate understanding and adjust instruction	L	B	L	H	H	Called on students to keep them on task
	Regulation	Excluded examples to save time and added examples to increase student understanding	L	B	L	L	B	Made no changes from original plans
Postactive	Evaluation	Evaluated goal accomplishment in terms of student understanding and content coverage	L	L	L	H	H	Evaluated goal accomplishment in terms of content coverage
	Suggestions	Gave ideas for better monitoring of students and clearer and more interesting instructional techniques	L	L	B	H	H	Gave ideas for better time management

H = Cognitions resembled those of teachers in Group HQ
L = Cognitions resembled those of teachers in Group LQ
B = Cognitions resembled those of teachers in both groups.

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Table 1
Lesson Dimensions and Dimension Indicators

Dimensions	Description of Dimension Indicators
Tasks	
Tools/Materials	Uses appropriate tools and materials to facilitate content clarity in an efficient manner.
Motivational Strategies	Uses tasks that capture students' curiosity and inspires them to speculate and to pursue their conjectures. The diversity of student interests and experiences must be taken into account
Content	Organization of content such that connections between mathematical concepts are made clear. Accuracy of mathematical content.
Difficulty Level	Uses tasks that are suitable to what the students already know and can do and what they need to learn or improve on. Appropriate modes of instruction must be selected based on the teachers' insights into the ways in which different students learn mathematics.
Sequencing	Sequences tasks such that students can progress in their cumulative understanding of a particular content area and can make connections among ideas learned in the past to those they will learn in the future.
Learning Environment	
Social/Intellectual Climate	Establishes and maintains a positive rapport with and among students by showing respect for and valuing students' ideas and ways of thinking.
Administrative Routines	Uses effective procedures for organization and management of the classroom so that time is maximized for students' active involvement in the discourse and tasks.
Instructional Routines	Uses classroom instructional structures that encourage and support student involvement as well as facilitate goal attainment.
Pacing	Provides and structures the time necessary for students to express themselves and explore mathematical ideas and problems.
Student Deportment	Enforces classroom rules and procedures to ensure appropriate classroom behavior.
Discourse	
Questioning	Poses variety of levels and types of questions using appropriate wait times that elicit, engage and challenge students' thinking.
Student Responses	Requires students to give full explanations and justifications or demonstrations orally and/or in writing.
Teacher-Student Interaction	Communicates with students in a non-judgmental manner and encourages the participation of each student.
Teacher Responses	Listens carefully to students' ideas and makes appropriate decisions regarding when to offer information, when to provide clarification, when to model, when to lead and when to let students grapple with difficulties.
Student-Student Interaction	Encourages students to listen to, respond to, and question each other so that they can evaluate and, if necessary, discard or revise ideas and take full responsibility for arriving at mathematical conjectures and/or conclusions.

Table 2
Components of Teacher Cognitions and Description of Indicators

Cognitions		Description of Indicators of Cognition
Overarching		
Knowledge	Pupils:	Has specific knowledge of pupils' prior knowledge and experiences, abilities, attitudes and interests.
	Content:	Has conceptual and procedural understandings of the content and is aware and appreciates the connections among it and past and future areas of difficulty.
	Pedagogy:	Has understanding of how students learn mathematics that guides them in developing suitable teaching strategies and anticipating and preparing for areas of difficulty.
Beliefs	Mathematics:	Views mathematics as a "dynamic and expanding system of connected principles and ideas constructed through exploration and investigation." (NCTM, 1991, p.133)
	Teacher:	Views their role as one of a facilitator of student learning through selections of problem-solving tasks and the leading and orchestration of communication in which students are challenged to think for themselves through mathematical reasoning.
	Students:	Views the role of their students as active participants in their own learning. They should make conjectures, propose approaches and solutions to problems, debate the validity of one another's claims, and verify, revise and discard ideas on the basis of their own and other students' mathematical reasoning.
Goals		To help students construct their own meaning so that they will develop conceptual, as well as procedural, understanding and will value the mathematics and feel confident in their abilities.
Preactive		
Planning	Objectives:	The focus of the lesson is on building conceptual understanding, based on what the students already know, and focusing on mathematical processes underlying the procedures to be developed, as well as the skill development required by the content cover specifications.
	Structure:	Tasks are logically sequenced to build on previous student understanding, and are appropriate for clarifying new concepts and arousing students' interest and curiosity.
	Phases:	Temporal boundaries that demarcate different segments of the lesson to be enacted: (1) Initiation (establishing students' readiness for learning); (2) Development (building new concepts) and; (3) Closure (integrating and extending new concepts).
Interactive		
Monitoring		Observes, listens to, and elicits participation of students on an ongoing basis in order to assess student learning and disposition toward mathematics.
Regulating		Adapts or changes instructional strategies while teaching based on the information received through monitoring student learning and interest.
Postactive		
Evaluating		Describes and comments on students' understanding of concepts and procedures and dispositions toward mathematics as well as the effects of their instruction on these outcomes.
Suggesting		Uses information from their evaluation of student learning and instructional practices to revise and adapt their subsequent plans for instruction.

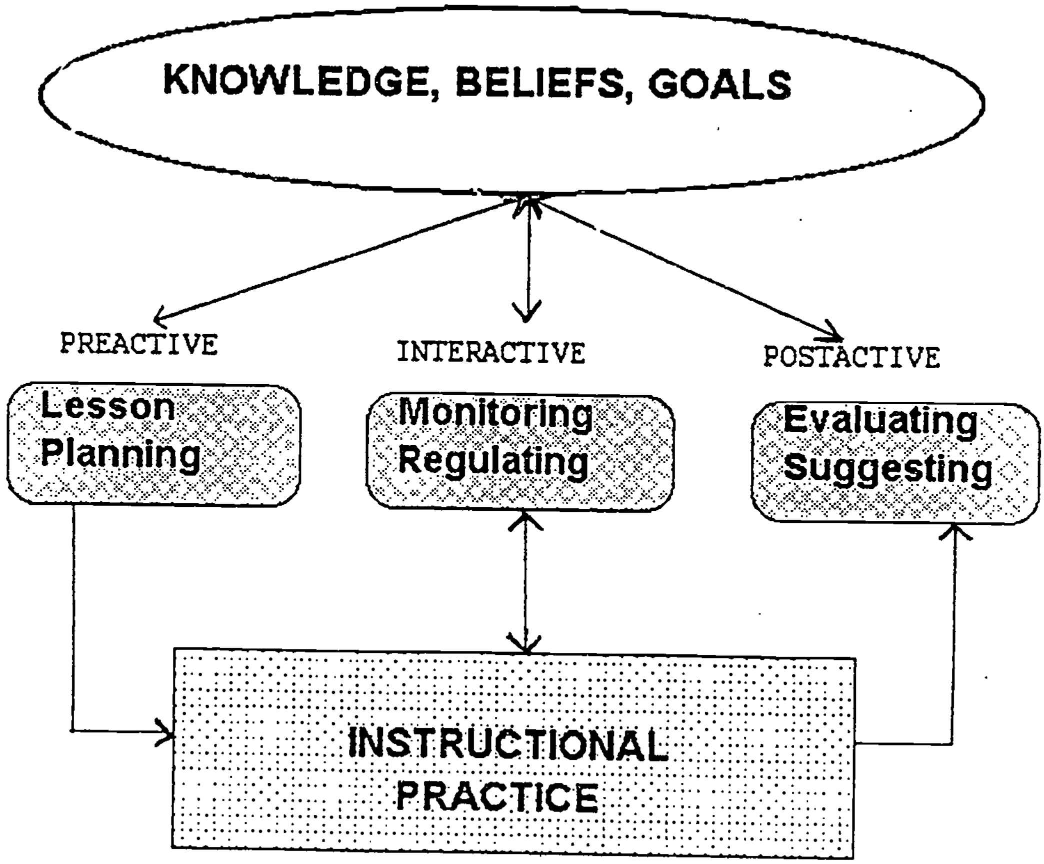

Figure 1. A model for the analysis of teacher cognitions related to instructional practice in mathematics